

# THE ANALYTIC EXAMINATION OF TIME DEPENDENT VARIANCE COMPONENTS

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## Introduction

The classical analysis of a nested design entails the estimation of the variance components as a means of understanding their contribution to total variation (1). In addition, a residual analysis is recommended as a way of verifying the assumption of the classical model (2). The use of the nested design in process study has a different intent and, thus, requires a different approach for analysis.

The intent of a nested design in process study is to tie the components of variance to sources of variation that may be present in a process and to provide a way of studying the data to understand how the effects of these sources of variation are manifested in process outputs. For example, a study of a batch process might consist of taking a fixed number of samples from a number of different batches. The intent of the data analysis would then be to provide managers, engineers, and operations personnel with information about the physical causes contributing to the variability within batches and between batches. For example, improper mixing might be one of the suspected sources contributing to variation within batches. Differences in procedures by different shifts who make the batches might contribute to between-batch variation. The analysis of the data should support an exploratory investigation of how the within batch and between-batch variation is behaving. Knowing that sources of variation act consistently or inconsistently support appropriate management, engineering, and operational actions.

An understanding of how within sources of variation are affecting the process is gained by plotting a measure of within batch variation on a control chart\*. Such a time ordered plot may be thought of as providing a "picture" of the process. From this "picture" it may be found that within batch variation is higher at some times than at others; or possibly that there is a cycle or trend in the within batch variation that requires an explanation. A commonly used model which is the basis of this analysis by control chart is a common cause/special cause model. Common cause sources in this instance would be those thought to affect the within batch variation of each and every batch. Special cause sources would be those that act intermittently to give different levels of variation within some batches as compared to others (4). A batch that shows evidence of being subject to special cause sources would be investigated to try to understand what sources of variation were present to give different levels of within batch variation. If only common cause sources of variation are affecting within batch variability, then improvement in the process will come by developing a further understanding of those common causes affecting all batches.

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\* Calculations for Control Chart construction may be found in Grant and Leavenworth (3).